# 15 Shapes with 96 Orientations

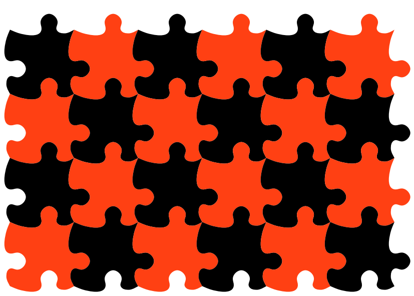
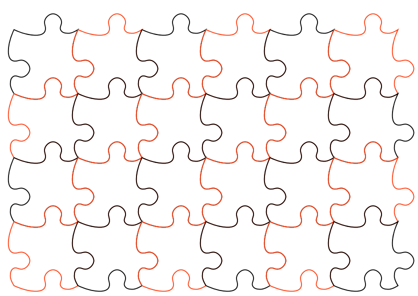
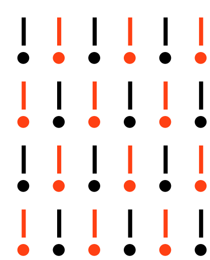
The TessiePuzzlePieces typefaces feature 15 distinct shapes of puzzle pieces. If the vertices of these shapes are connected with straight lines, the result will be squares. Further, all the edges are shaped in the same way. They differ because the common edge can be rotated or flipped.

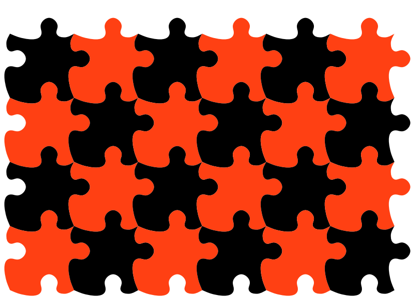
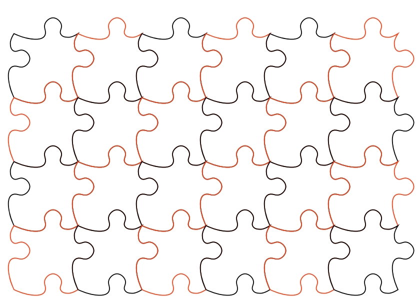
Each distinct shape can be rotated 90°, 180°, and 270° to form of set of four orientations. Each of these can in turn be flipped, yielding four more. So there are 15*8=120 possible orientations. However, four of the shapes have mirror symmetry and two of them have two-fold rotational symmetry, so these six shapes have only four different orientations. Hence, 120 possible orientations less 24 that are duplicates yields 96 different orientations for the 15 distinct shapes. The standard keyboard can access 94 characters, so two of the orientations are on accented characters.

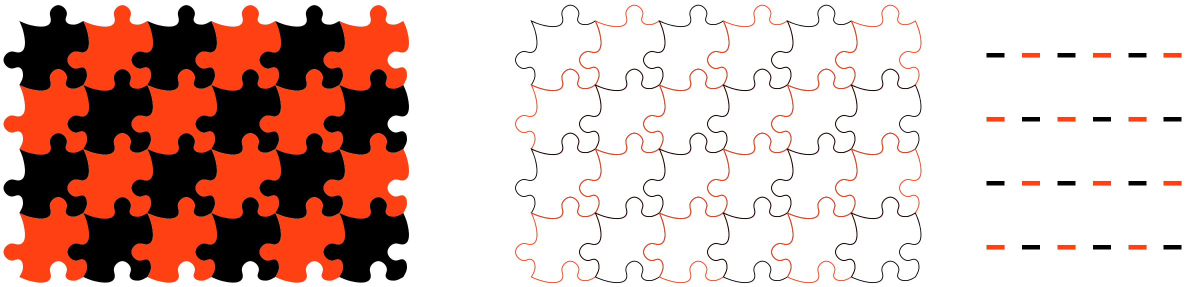
For convenience going forward, here is a listing of the 15 distinct shapes with the keys that contain them. A expanded version of this listing is at the end of this document.

Group 1 !"#$%&'(
Group 2 )*+,
Group 3 -./0
Group 4 12345678
Group 5 9:;<=>?@
Group 6 ABCDEFGH
Group 7 IJKLMNOP
Group 8 QRST
Group 9 UVWXYZ[\
Group 10 ]^_`
Group 11 abcd
Group 12 efghijkl
Group 13 mnop
Group 14 qrstuvwx
Group 15 yz{|}~ÄÅ

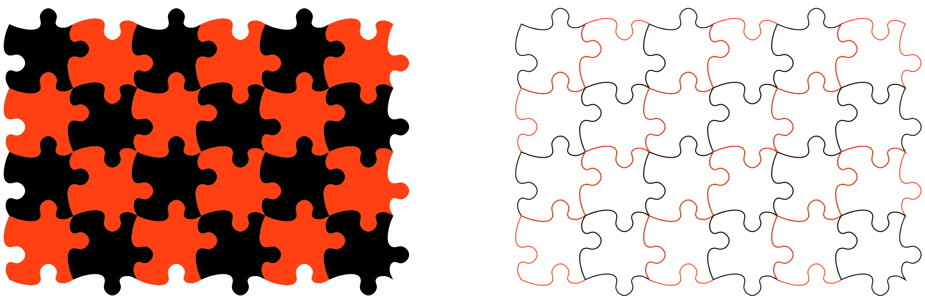
The simplest way for a shape with vertices of a square to tile is with translation. The right edge is copied and moved to the left edge and the top edge is copied and moved to the bottom edge. Three of the groups (1, 2, and 3) are formed in this way and tile with a single orientation. Below are examples of each. Tessellation fans will recognize these as Heesch type TTTT tilings.

There are numerous ways to tile with two orientations of a single shape. Below are examples of groups 4, 6, 7, & 9 (TGTG types):



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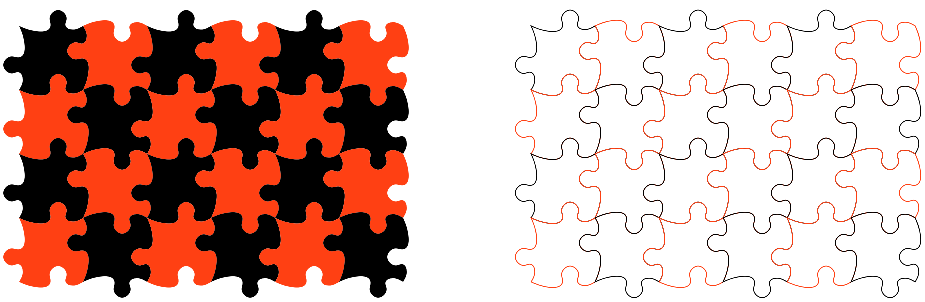
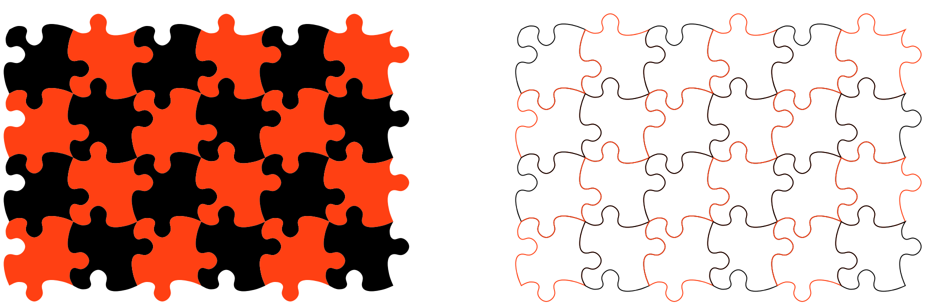
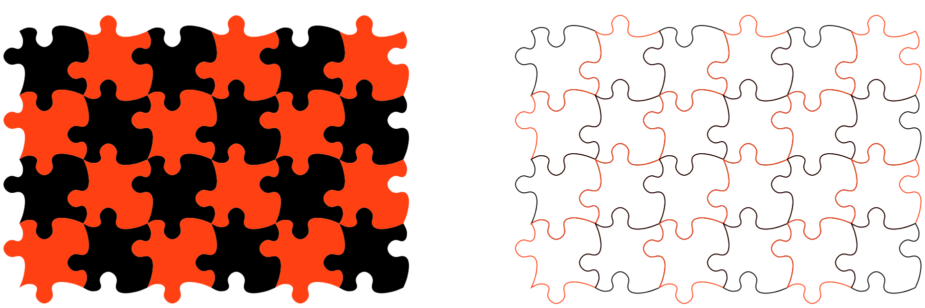


AGAGAG
AGAGAG
AGAGAG
AGAGAG

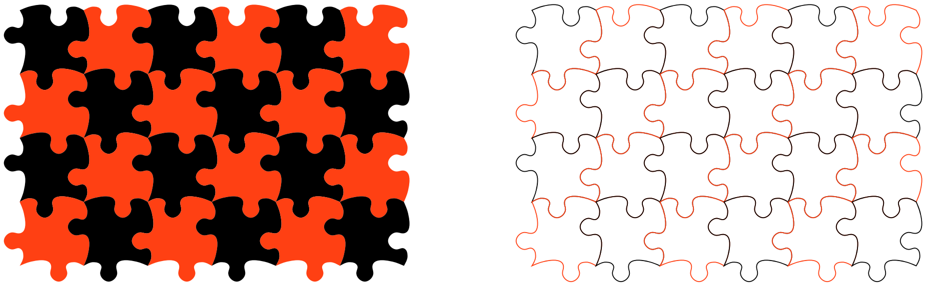


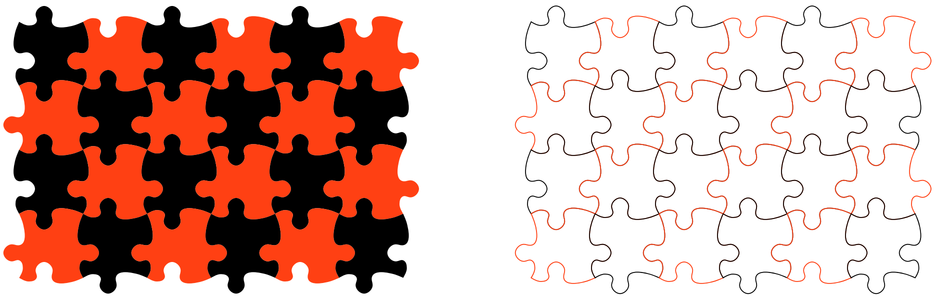
LNLNLN
LNLNLN
LNLNLN
LNLNLN

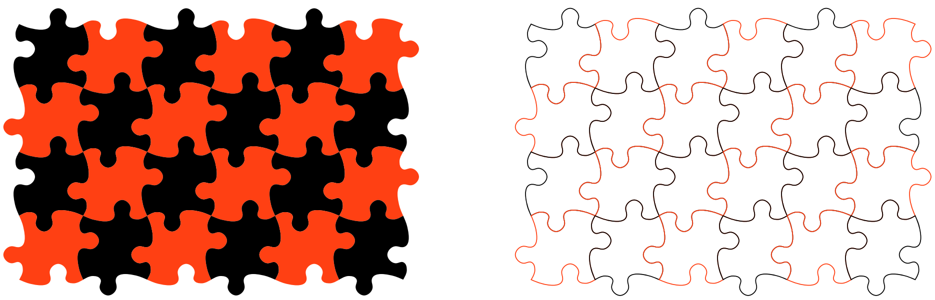


WYWYWY
WYWYWY
WYWYWY
WYWYWY

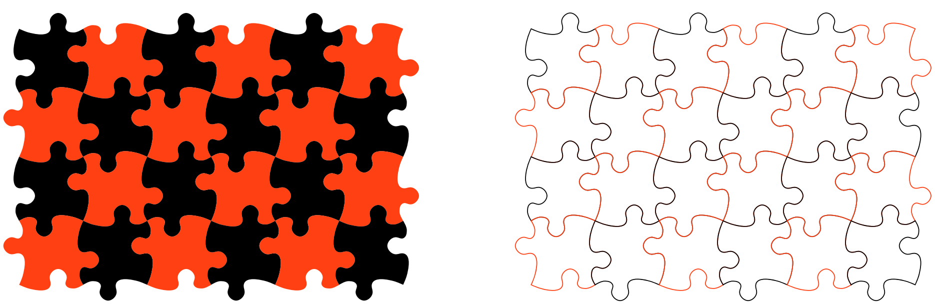
Groups 5, 12, & 13 can tile as G1G1G2G2 types:



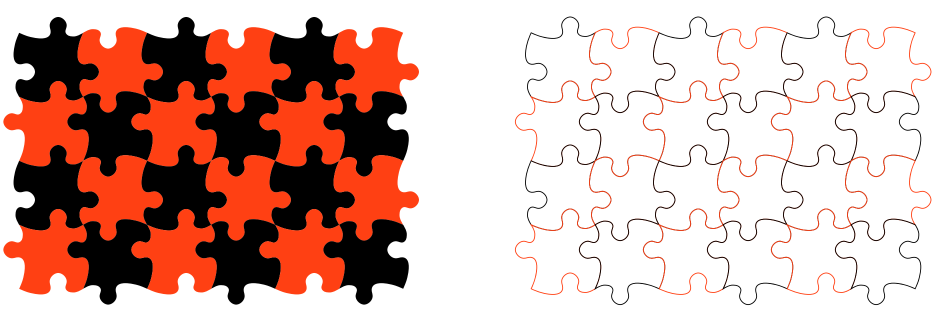



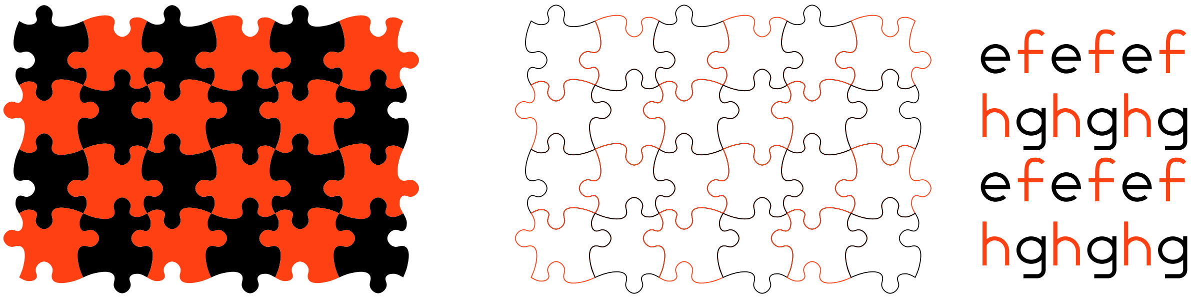




Group 10 tiles as type C4C4C4C4. Normally type C4C4C4C4 has a translation block of four but the tile or shape has two-fold rotational symmetry, reducing the block to two.

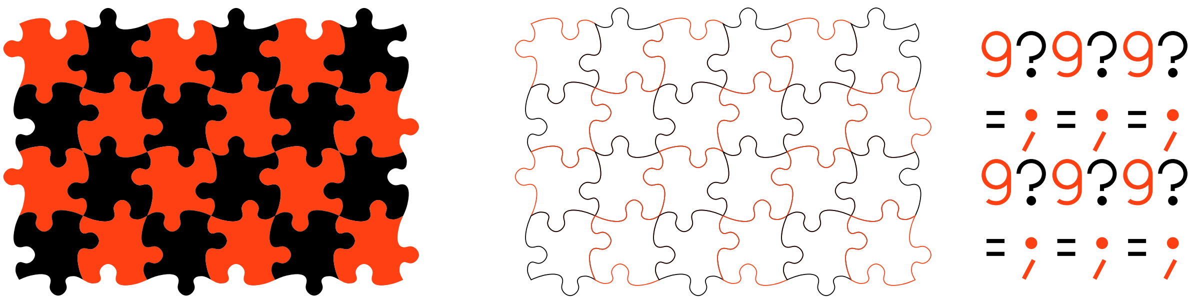



There are also numerous ways to tile with four orientations of a single shape. Groups groups 1, 8 tile as type C4C4C4C4.

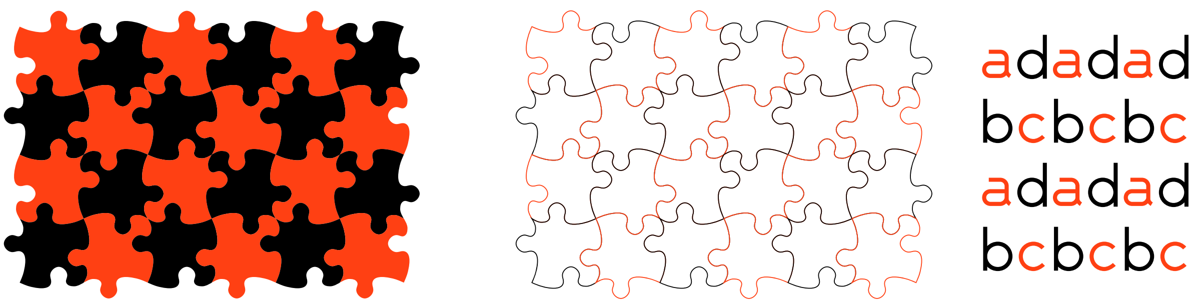


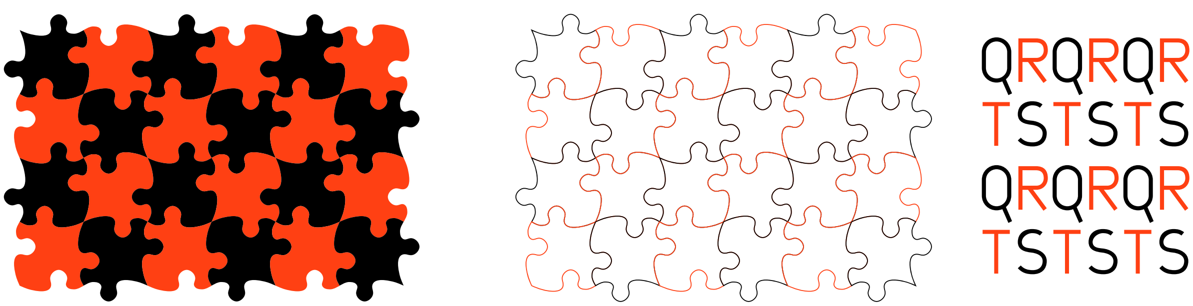



Group 5 tiles as type G1G2G1G2.

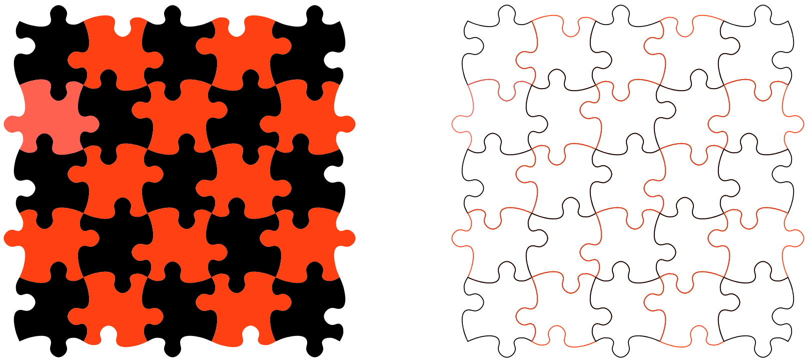



Groups 11 & 12 tile as both C4C4C4C4 and G1G2G1G2. The mirror symmetry of the shape allows them to satisfy both types simultaneously..




An isohedral tiling is one in which edges meet consistently throughout the tiling. If the edges are labeled A, B, C, and D, then if edge A is adjacent to edge B in one place in the tiling, it is adjacent to edge B everywhere. If sometimes edge A is adjacent to edge B and sometimes it is adjacent to edge D, the tiling is anisohedral. Groups 14 and 15 do not tile isohedrally but they do tile anisohedrally. Below are examples for both. There are also other anisohedral tilings of these groups.



tsxut

sxuts

xutsx

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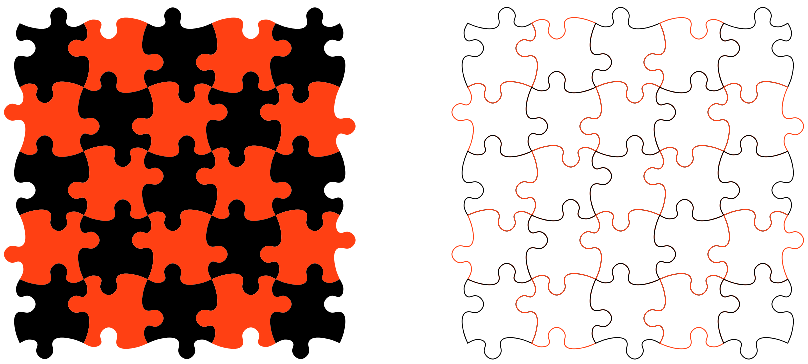
tsxut



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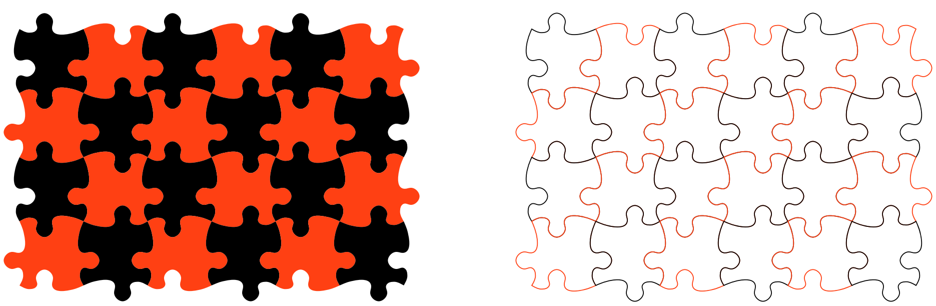
The tiles of groups 14 and 15 are anisohedral tiles because they do not tile isohedrally, only anisohedrally. Some of the other groups will also tile anisohedrally in addition to tiling isohedrally. Here are three examples, one of group 12 and two of group 5.

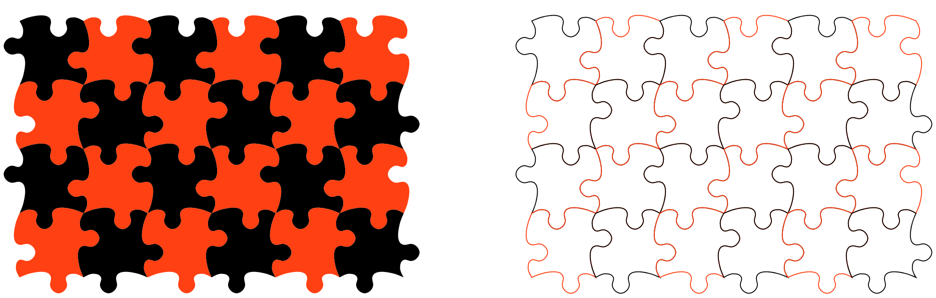


efefef

jijiji

efefef

jijiji



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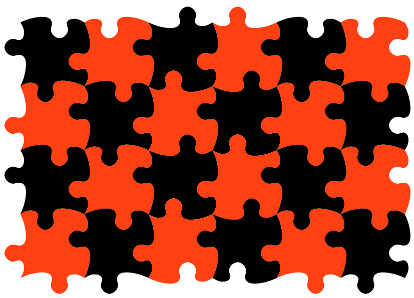
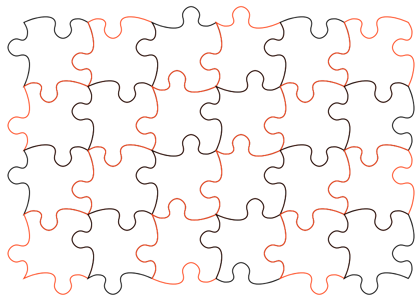
=:=:=:





9><?9>
>9?<>9
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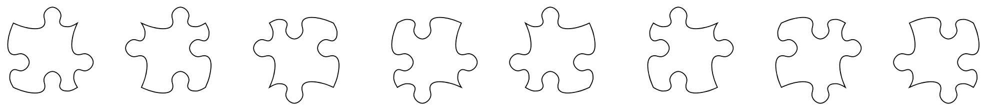
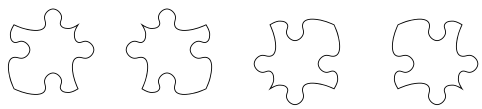
Group 1



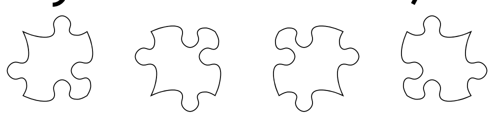
! " # $ % & ' (

Group 2



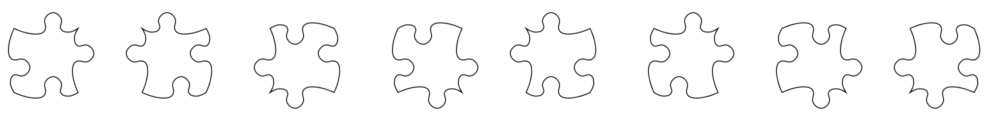
) * + ,

Group 3



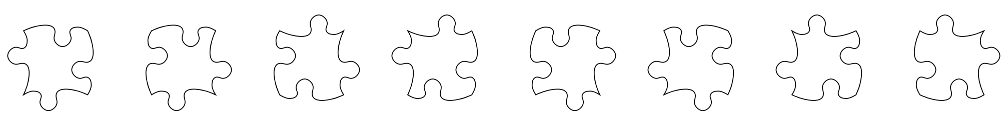
- . / 0

Group 4



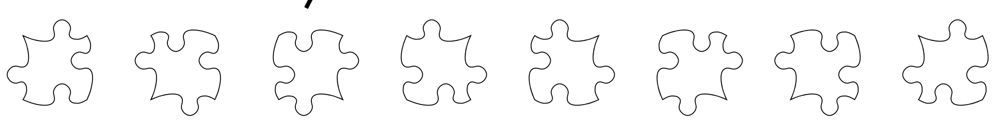
1 2 3 4 5 6 7 8

Group 5



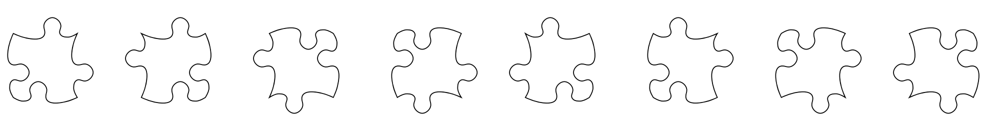
9 : ; < = > ? @

Group 6



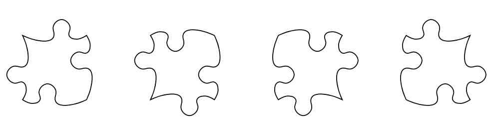
A B C D E F G H

Group 7

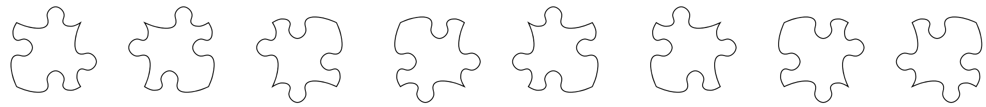


I J K L M N O P

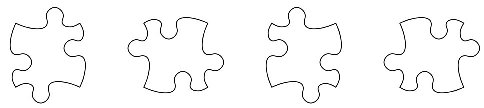
Group 8



Q R S T

Group 9 

U V W X Y Z [ \

Group 10 

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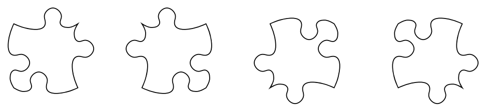
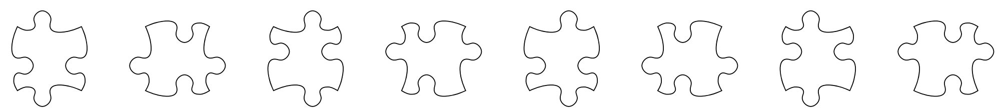
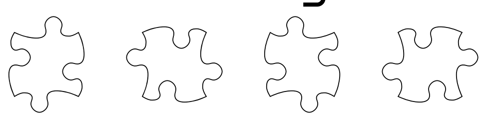
Group 11 

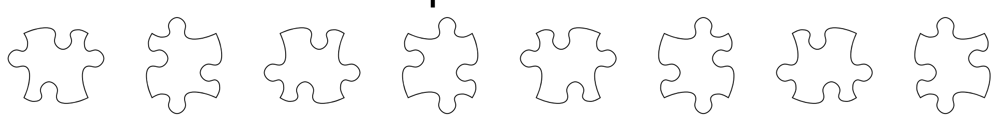
a b c d

Group 12 

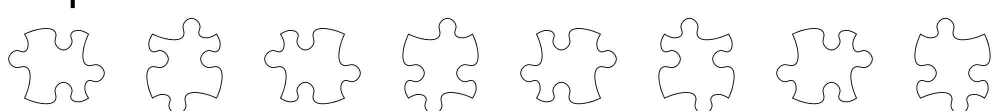
e f g h i j k l

Group 13 

m n o p

Group 14 

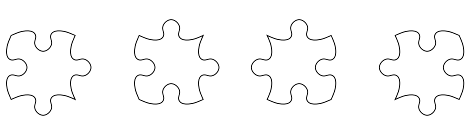
q r s t u v w x

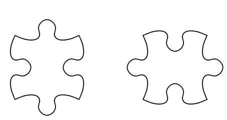
Group 15 

y z { | } ~ Ä Å

**Addendum**

If the asymmetric edges are replaced with identical edges that mirror around their centerpoint, the 15 distinct shapes are reduced to two:



¡ ™ £ ¢ and 

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The first set tiles in four ways and the second in one.